Entropy-based implied volatility and its information content*

Xiao Xiao†[1,2,3] and Chen Zhou ‡[1,2,4]

[1]Erasmus School of Economics, Erasmus University Rotterdam

[2]Tinbergen Institute, The Netherlands

[3]Duisenburg School of Finance

[4]De Nederlandsche Bank

Abstract

This paper investigates the maximum entropy approach on estimating implied volatility. The entropy approach also allows to measure option implied skewness and kurtosis nonparametrically, and to construct confidence intervals. Simulations show that the entropy approach outperforms the Black-Scholes model and model-free method in backing out implied volatility, when the risk neutral distribution of the underlying asset deviates from log-normal distribution, and when the number of available options is limited. Using S&P500 index options, we apply the entropy method to obtain implied volatilities and their confidence intervals. We find that the entropy-based implied volatility subsumes all information in the Black-Scholes implied volatility and historical volatility. In addition, it has more predictive power than the model-free implied volatility, in both in-sample and out-of-sample setup. Entropy-based variance risk premium performs better than other alternatives in predicting future monthly market return in both in-sample and out-of-sample.

JEL Classification: C14, G13, G17

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†Department of Economics, Erasmus University Rotterdam, P.O.Box 1738, 3000DR, The Netherlands. Email: xiao@ese.eur.nl.

‡Department of Economics, Erasmus University Rotterdam, P.O.Box 1738, 3000DR, The Netherlands. Economics and Research Division, De Nederlandsche Bank, 1000AB Amsterdam, The Netherlands. Email: c.zhou@dnb.nl, zhou@ese.eur.nl.

1 Introduction

Since option prices reflect investors' expectation on the future movements of the underlying asset price, risk measures implied in the cross-section of option prices across the strike prices are considered to be informational superior to their historical counterparts. Many studies investigate information content, dynamic properties and asset pricing implications of different option implied risk measures, such as Black-Scholes implied volatility (BSIV, Black and Scholes (1973)), model-free implied volatility, model-free implied skewness and model-free implied kurtosis [1]. For instance, several studies find implied volatility to be informationally superior to the historical volatility of the underlying asset[2]. Neumann and Skiadopoulos (2013) show that the dynamics of the higher order implied moments can be statistically forecasted. DeMiguel et al (2013) present evidence that option-implied information can improve the selection of mean-variance portfolios and the out-of-sample performance of the portfolio.

Although the option implied risk measures have been shown to be informationally useful, it is unclear whether these measures can actually represent the characteristics of the true underlying risk neutral distribution, when the distribution deviates from the assumption of the model which derives the measures. The most widely-used option implied risk measure, the Black-Scholes implied volatility, is calculated from the Black-Scholes formula such that the model is consistent with the option price observed in the market. Whereas the underlying stock can have only one volatility for different strike prices, graphing B-S implied volatilities against strike prices for a given expiry yields a skewed "smile" instead of the flat surface. Furthermore, Neumann and Skiadopoulos (2013) show that the risk neutral skewness calculated from S&P500 index options is consistently negative and the implied kurtosis is always larger than 3 over from 1996 to 2010. Both evidence point to the fact that the empirical distribution observed in the financial market is inconsistent with the assumption in the

[1] Dennis and Mayhew (2002), Jiang and Tian (2005), Bali and Murray (2013), DeMiguel et al (2013), and Neumann and Skiadopoulos (2014)

[2] See Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Christensen and Prabhala (1998), Fleming (1998), and Blair, Poon, and Taylor (2001). Busch, Christensen, and Nielsen (2008) find evidence in bond, stock and exchange rate markets.

Black-Scholes model that the asset return under the physical measure and the risk neutral measure follows a lognormal distribution. When the risk neutral or the physical distribution deviates from lognormal, the BS implied volatility may not be an accurate estimate of the risk neutral volatility.

Unlike traditional notion of implied volatility, Britten-Jones and Neuberger (2000) and Bakshi and Madan (2003) propose a model-free implied volatility (MFIV) which is independent of option pricing models. It is derived entirely from no-arbitrage conditions and can be considered as a linear combination of European call and put option prices with strikes spanning the full range of possible values for the underlying asset at maturity. However, the relationship only exactly holds under the diffusion assumption. Under the stochastic volatility and random jump (SVJ) model, Jiang and Tian (2005) show the truncation error and the discretionary error of the MFIV. Although the estimation error is admissible under certain conditions, it tends to be larger when the underlying distribution is more negatively skewed, when the available number of options is limited and when the market is more volatile.

In this paper, we apply the principle of maximum entropy to estimate implied volatility (ETIV) from option prices. There are several advantages of this method. First, this approach inherits the merit in model-free method, which does not depend on any parametric model and lets the data determine the shape of the distribution. Second, unlike the model-free implied moments in Bakshi and Madan (2003), the proposed method does not require a large number of options with strike prices covering the entire support of the return distribution. Instead, this method can produce accurate estimates of option implied volatility with limited number of options. Third, implied skewness (ETIS) and implied kurtosis (ETIK) can also be estimated using this method. Last but not least, this method allows construction of confidence intervals for the implied volatility, since a nonparametric analog of likelihood ratio statistics proposed by Kitamura and Stutzer (1996) can be applied in our case.

We consider 4 scenarios of risk neutral distributions and different number of available options to compare the performance of entropy method, model-free method and Black-Sholes model in backing out option implied risk measures. We first calculate option prices by numerical integration under different risk neutral distributions, and then calculate implied moments using different methods. We find that when the risk neutral distribution deviates from lognormal distribution, with heavy tail and negative skewness, ETIV is closer to the true value than BSIV and MFIV. When we reduce the number of available options or increase the true volatility, the estimation error of MFIV becomes more salient while ETIV estimates remain robust under different specifications. We also compare the performance of implied skewness and implied kurtosis by ET (ETIS and ETIK) and by model free method (MFIS and MFIK). We find that when the underlying risk neutral distribution deviates from lognormal, all measures underestimate the risk neutral skewness and kurtosis, but ETIS and ETIK are slightly closer to the true value than MFIS and MFIK. In addition, we provide the confidence interval of ETIV and the coverage ratio under different distributions.

In the empirical study, we calculate the entropy-based implied moments and model-free implied moments from S&P500 index options from 1996 to 2013. Moreover, we examine the information content of ETIV, MFIV and BSIV in predicting realized volatility in the next month. Our results suggest that ETIV subsumes all information in BSIV and historical volatility and it has higher forecasting ability than MFIV. In the out-of-sample analysis, ETIV continues to provide superior forecasts and performs the best in high volatility regime. We also find evidence that MFIS and MFIK tend to underestimate the risk neutral skewness and kurtosis.

Our approach is closely related to the application of the principle of maximum entropy in Buchen and Kelly (1996), who find the distribution estimated by this method is able to accurately fit a known density, given simulated option prices at different strikes. In this paper, we conduct more comprehensive analysis on this method by considering more realistic specifications and providing suggestions on how to apply the method empirically. In an important departure from Buchen and Kelly (1996), we focus on the implied moments and compare the estimation error and forecasting ability with their model-free counterparts and provide confidence intervals for the implied volatility. This study is also related to Stuzter (1996). Instead of pricing options using maximum entropy distribution, we conduct the reverse procedure to back out information from option prices. Our contribution is three-fold. First, we propose a new set of estimators for risk neutral volatility, skewness and kurtosis, which are more accurate than their model-free counterparts in certain circumstances. Second, to the best of our knowledge, this paper is the first to construct confidence intervals of implied volatility based on maximum entropy principle, from both the simulated data and traded option data. Third, using market prices of S&P500 index options, we provide both in-sample and out-of-sample evidence that ETIV has better forecast performance than other implied volatility measures.

The remainder of the paper proceeds as follows. Section 2 illustrates the estimation of option implied risk measures using maximum entropy principle. Section 3 compares accuracy of different implied moments estimator under different risk neutral distributions of the underlying asset and shows the confidence interval for ETIV. The forecasting ability of information content of different implied volatilities are investigated using nonoverlapping samples in Section 4 and Section 5 concludes.

2 Estimation of option implied risk measures using maximum entropy principle

In this section, we first introduce the maximum entropy principle in the context of option pricing and then illustrate how to back out risk neutral distribution from option prices nonparametrically. From the estimated risk neutral distribution, option implied risk measures, i.e. volatility, skewness and kurtosis can be calculated accordingly. One striking feature of this method is the ability to construct confidence intervals for these risk measures. The details of constructing confidence intervals for implied volatility is explained in (2.2).

2.1 Maximum entropy principle in the context of risk neutral pricing

The absence of arbitrage guarantees the existence of a risk-neutral probability measure under which the price of any security is the expectation of discounted payoffs. In the following paragraphs we seek to characterize this probability measure. Thus all probabilistic statements refer to risk-neutral probabilities rather than objective probabilities, unless otherwise specified.

Let X_t be a random variable that represents the gross return of a stock at fixed expiry time t in the future and S_0 is the current price of the asset. The value of the call option with strike price K at time 0 is the expectation of the discounted payoff at time t under the risk neutral measure Q:

$$C = E^Q[max(S_0X_t - K, 0)]/r_t, \tag{1}$$

where r_t is the gross risk free rate from time 0 to t. In a discrete state setting, assume that there are n possible states for X_t and q_1, ..., q_n are probabilities affiliated to states X_{t1}, ...X_{tn}. For q_i to present a valid density, we require $q_i > 0$ and $\sum_{i=1}^n q_i = 1,\ i = 1, ..., n$. Under the risk neutral pricing framework, the prices of call options and put options are:

$$C = \sum_{i=1}^n q_i(max(S_0X_{ti} - K, 0))/r_t,\ \ P = \sum_{i=1}^n q_i(max(K - S_0X_{ti}, 0))/r_t. \tag{2}$$

where C and P are the call and put option prices, q_i is the risk neutral probability affiliated to state X_{ti} and K is the exercise price of the option.

Given the fact that the number of possible states is much larger than the number of available options, it is not sufficient to uniquely determine the underlying distribution of the asset. However, Buchen and Kelly (1996) show that if these option prices used to constrain the distribution has maximum entropy, then a unique distribution is obtained. Since entropy measures the amount of missing information, the maximum entropy distribution is the least prejudiced compatible with the given constraints, in the sense that it is least committal with respect to this missing information. From the viewpoint of statistical inference, there is no reason to prefer any other distribution (Buchen and Kelly (1996)). The entropy of the distribution of X_t is defined by:

$$\ell_{ET} = -\sum_{i=1}^n q_i log(q_i). \tag{3}$$

According to the principle of maximum entropy, we select the q_i such that entropy of the distribution is maximized subject to the constraints. The constraints are based on the risk neutral pricing formula of call and put options:

$$C(j) = \sum_{i=1}^n q_i(max(S_0X_{ti} - K_c(j), 0))/r_t,\ j = 1, ..., k_1 \tag{4}$$

$$P(j) = \sum_{i=1}^n q_i(max(K_p(j) - S_0X_{ti}, 0))/r_t,\ j = 1, ..., k_2 \tag{5}$$

$$\sum_{i=1}^n q_i = 1,\ q_i \geq 0,\ \ k_1 + k_2 = k. \tag{6}$$

where k_1 is the number of call option prices, k_2 is the number of put option prices, and X_{t1}, ...,X_{tn} are possible states of the gross stock return. It can be considered as a Lagrange problem, in which we estimate the risk neutral probabilities $(q_1, ..., q_n)$ given option prices and possible states of the stock return. The value of $(q_1, ..., q_n)$ that maximizes ℓ_{ET} under the k constraints is also called the exponential tilting estimator and denoted by $(\hat{q}_1, ..., \hat{q}_n)$. To present the constraints in a concise manner, we express the first k constraints as:

$$\sum_{i=1}^{n} q_i g_j(X_{ti}) = 0, \; j = 1, ...k. \tag{7}$$

The Lagrange function associated with the constrained optimization problem can then be formulated as:

$$\mathcal{L} = \sum_{i=1}^{n} q_i log(q_i) + \gamma(\sum_{i=1}^{n} q_i - 1) + \lambda'(\sum_{i=1}^{n} q_i g(X_{ti})), \tag{8}$$

where $\gamma \in \mathbb{R}$ and $\lambda \in \mathbb{R}^m$ are Lagrange multipliers. $g(X_{ti})$ is a k dimensional vector $[g_1(X_{ti}), ..., g_k(X_{ti})]$. It is straightforward to show that the first order conditions for $\mathcal{L}$ are solved by:

$$\hat{q}_i = \frac{exp(\hat{\lambda}' g(X_{ti}))}{\sum_{i=1}^{n} exp(\hat{\lambda}' g(X_{ti}))}, \; i = 1, ..., n \tag{9}$$

$$(\hat{\lambda}_1, ..., \hat{\lambda}_k) = \arg min \sum_{i=1}^{n} exp(\lambda' g(X_{ti})), \tag{10}$$

where λ_j is Lagrange multiplier of the jth constraint. We can see that the estimated $\hat{q}_i$ is presented as a function of the Lagrange multipliers and the Lagrange multipliers can be solved from minimizing a strictly convex function. Instead of estimating n unknown probabilities (q_i, ..., q_n), we only have to obtain the estimates of k Lagrange multipliers. It can be easily proved that a unique solution of $\hat{\lambda}_j$ exists due to strict convexity of the function. In the empirical analysis, we find that the convergence of $\hat{\lambda}_j$, $j = 1, ..., k$ is fast and reliable.

Given the estimated risk neutral probabilities and associated possible states, the entropy based implied volatility (ETIV), skewness (ETIS) and kurtosis (ETIK) are calculated as:

$$ETIV = V^Q = \sqrt{\sum_{i=1}^{n} \hat{q}_i(log(X_{ti}) - \mu^Q)^2},\ \mu^Q = \sum_{i=1}^{n} \hat{q}_i log(X_{ti}) \tag{11}$$

$$ETIS = \sum_{i=1}^{n} \hat{q}_i((log(X_{ti}) - \mu^Q)/V^Q)^3, \tag{12}$$

$$ETIK = \sum_{i=1}^{n} \hat{q}_i((log(X_{ti}) - \mu^Q)/V^Q)^4. \tag{13}$$

Note that when it refers to implied volatility, i.e implied volatility calculated by Black Scholes model, it is based on continuous compounded stock returns, especially when one purpose of this study is to compare BSIV with ETIV. Hence the three measures are calculated from $\log(X_i)$ rather than X_i. In the empirical study, we specify the possible states $X_{t1}, ...X_{tn}$ to be a equally distant series. For instance, we assume the possible state of gross monthly stock return to be 0.7 to 1.3 with step 0.001.

We note that entropy measure is one member in the Cressie-Read divergence family, which is also called Kullback-Leibler divergence. Apart from that, entropy estimator is included as a special case in a class of generalized empirical likelihood estimators. It can be argued that other members in the family, i.e. empirical likelihood (EL) or Euclidean divergence, may perform the same or even better because the estimators in the family share the common structure and possess the same asymptotic variance. However, we find some advantages of entropy measure compared to other measures in the simulation and empirical study. First, unlike Euclidean estimators, entropy method performs robust to different specifications of possible states. To be more specific, whether we simulate states from a certain distribution, or enforce a equally distant series as states, the results of the estimated risk neutral distribution do not change as long as the states cover the range of the strike prices, which means the distribution is solely determined by the option prices rather than the given states. Second, we prefer entropy to EL because EL's implied probability exhibit questionable behavior, by placing a large weight on a few extreme observations. Theorem 1 in Schennach (2007) shows that under the unbounded moment conditions, the slightest amount of misspecification can cause the first-order asymptotic properties of EL to degrade catastrophically. To back out the risk neutral distribution, the distribution which generates the option prices and the given distribution of the stock return states typically do not coincide, which causes model misspecification. In addition, when we construct confidence interval for implied volatility, one of the constraint is unbounded. Under these circumstances, we are in favor of entropy measure in this paper.

2.2 Confidence interval of entropy-based implied volatility

After estimating the risk neutral probabilities (q_i, ..., q_n), the option implied risk measures $ETIV$, $ETIS$ and $ETIK$ can be easily calculated. A nice feature of entropy method is that it facilitates the construction of confidence interval. Different from the Black-Scholes implied volatility (BSIV) and model-free implied volatility (MFIV), likelihood-ratio based confidence regions can be constructed for ETIV using likelihood-like ratio statistics. Suppose we wish to construct tests of the following restrictions: $H_0: \ V^Q = \hat{V}^Q$, a nonparametric analog of the parametric likelihood ratio statistics is proposed by Kitamura and Stutzer (1997):

$$LR_T = 2n[\log M(\hat{V}^Q) - \log M(\hat{V}_c^Q)] \xrightarrow{d} \chi_1^2 \tag{14}$$

where $M(\hat{V}^Q) = \frac{1}{n}\sum_{i=1}^{n} exp(\hat{\lambda}' g(X_i))$, and $\hat{\lambda}$ is the solution of the following problem:

$$(\hat{\lambda}_1, ..., \hat{\lambda}_k, \hat{\lambda}_{k+1}) = \frac{1}{n} \arg min \sum_{i=1}^{n} exp(\lambda' g(X_{ti})), \tag{15}$$

Note that there are $k+1$ constraints including the constraint $V^Q = \hat{V}^Q$. Since the likelihood-like ratio test statistics obeys the chi-square distribution with 1 degree of freedom, the confidence interval for $ETIV$ can be constructed by moving the volatility constraint around the null hypothesis until the difference between the unconstrained and constrained likelihood ratio statistics is larger than χ_1^2. To construct the confidence interval for $ETIV$, we implicitly assume that the mean of the risk neutral distribution of the continuous compounded return μ^Q is fixed when moving the constraint of V^Q. It can be easily derived that if we include at-the-money call and put option prices as constraints where $K_c = K_p = S_0$, the mean of the discrete stock return under the risk neutral measure is derived as:

$$\sum_{i=1}^{n} q_i X_{Ti} = \frac{(C_{atm} - P_{atm})r_t + 1}{S_0}, \tag{16}$$

where C_{atm} and P_{atm} are at-the-money call and put option prices. To ease the computational burden and to guarantee convergence of the problem, we make an approximation that μ^Q, the mean of continuous log return under the risk neutral measure, is also fixed at $\sum_{i=1}^{n} \hat{q}_i log(X_{Ti})$ when we move the volatility constraint. In the simulation, we find that the means of the discrete return and continuous return under the risk neutral return are indeed close with the magnitude of 10^{-3}.

3 Performance of entropy-based option implied risk measures under different distributions

In this section, we compare the performance of entropy-based method, Black-Scholes model and model-free method in backing out implied risk measures from option prices. Entropy-based method is shown to be more accurate than Black-Scholes model and model-free method under heavy tail and non-zero skewness cases.

To investigate the capability of entropy-based method to back out implied volatility from option prices, we evaluate four different risk neutral distributions of the underlying continuous returns. To begin with, we consider the case in Black-Scholes model that stock price follows a geometric Brownian motion under the risk neutral measure:

$$dS_t = rS_t dt + \sigma S_t dw_t, \tag{17}$$

where S_t is the stock price at time t, r is the risk-free rate, σ is the constant instantaneous volatility of the process and dw_t is the increment in a standard Wiener process. Under this model, the risk neutral distribution of continuously compounded T-year returns $ln(R_T)$ is normally distributed:

$$ln(R_T) \sim N((r - \frac{1}{2}\sigma^2)T, \sigma^2 T) \tag{18}$$

We also consider the cases deviate when the continuous returns under the risk neutral measure follow a Student t distribution or Skewed Student t distribution. Skewed Student t distribution ($skewt(\eta, \lambda)$) suggested in Hansen (1994) has a mean of zero and a unit variance with degree of freedom η and skewness parameter λ. We consider two skew-t distribution with $\eta = 5$, $\lambda = -0.3$ and $\eta = 5$, $\lambda = -0.7$. The former is slightly asymmetric and the latter one is more negatively skewed. The continuously compounded T-year return $ln(R_T)$ is distributed as:

$$ln(R_T) \sim (r - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}\, skewt(\eta, \lambda). \tag{19}$$

The call and put option prices with strike price K and maturity T are calculated by numerical integration:

$$C(K,T) = \int_{K/S_0}^{\infty} (S_0 R_T - K) f(R_T) dR_T / r_T \tag{20}$$

$$P(K,T) = \int_{0}^{K/S_0} (K - S_0 R_T) f(R_T) dR_T / r_T \tag{21}$$

where $f(R_T)$ is the density function of R_T. The density function of log-skew t distribution is provided in the appendix. We specify the mean of the risk neutral distribution as $(r - \frac{1}{2}\sigma^2)T$ to ensure that the expectation of R_T is e^{rT} under the risk neutral measure. This relation holds exactly for log-normal distribution and approximately for the log-skewed t distribution.

The calculation of option prices employs annual risk-free rate r of 5%, annual volatility σ of 20% (40%) and initial stock price S_0 equal to 100. We calculate the call and put option prices using numerical integration for several moneyness and 5 maturities from one month to a year. Following Bakshi and Madan (2003), we only consider out-of-the money options and at-the-money options due to the liquidity reason in the option market. Moreover, in-the-money call and put option prices can be derived from put-call parity under the no arbitrage condition and hence the information in them does not add much value in simulation. We also consider different number of available options: call and put options with 7 or 3 pairs of strike prices. In the first case, we specify different set of strike prices to capture similar portion of the distribution for different maturities. In the second case, we reduce the number of available options and keep the strike prices constant for different maturities. The strike prices are presented in table 1 and 2 and the calculated option prices in each moneyness and maturity category under different distributions are reported in table 3 and 4.

Following the procedure illustrated in Section 2, we calculate ETIV, ETIS and ETIK using different pairs of options. To evaluate the performance of ETIV, we compare with

Black-Scholes implied volatility (BSIV) and model-free implied volatility (MFIV) proposed by Bakish and Madan (2003):

$$MFIV = E^Q[R_T^2] = e^{rT}\int_S^\infty \frac{2(1-\ln[\frac{K}{S_0}])}{K^2}C(K,T)dK + e^{rT}\int_0^S \frac{2(1+\ln[\frac{S_0}{K}])}{K^2}P(K,T)dK \tag{22}$$

In the discrete setting, the measure can be approximated as:

$$E^Q[R_T^2] \approx e^{rT}\sum_{i=1}^{m} \frac{2(1-\ln[\frac{K_i}{S}])}{K_i^2}C(K_i,T)\Delta K + e^{rT}\sum_{j=m+1}^{n} \frac{2(1+\ln[\frac{S}{K_j}])}{K_j^2}P(K_j,T)\Delta K \tag{23}$$

where $K_1, ..., K_m > S$, $K_{m+1}, ..., K_n < S$ and $\Delta K = (K_{max} - K_{min})/(n+1)$. Given limited number of options, we apply curve-fitting method to implied volatilities. Option prices are first translated into implied volatilities using the BS model. A smooth function is then fitted to the implied volatilities. The BS model is then used again to translate the extracted implied volatilities into call prices. Following Bates (1991) and Jiang and Tian (2005), we use cubic spines in the curve-fitting. For options with strike prices beyond the available range, we use the end-point implied volatility to extrapolate their option values.

To be comparable with ETIV and MFIV, BSIV estimates are calculated as the average of Black-sholes implied volatilities from all options. We report different IV estimators in different specifications in table 5 to table 8. The first column in table 5 represents four distributions and the second column shows different method. From table 5, we find that when the discrete return follows normal distribution, both BSIV and MFIV are the same as true value, while ETIV has small error when the maturity increases. However, when the underlying distribution exhibits heavy tail, Black Scholes formula cannot produce unbiased estimates of implied volatility. When the underlying distribution becomes more negative skewed, the estimates of BSIV becomes further away from the true value. MFIV performs better than BSIV under the four distributions, but the estimation error increases when the underlying distribution becomes more heavy tail and negatively skewed. When the underlying distribution becomes heavy tailed and negatively skewed, ETIV estimates are closer to the true value than both BSIV and MFIV. When we decrease the available number of options (Table 6 and 8) or increase the true volatility from 0.2 to 0.4 (Table 7 and 8), the advantage of ETIV compared to MFIV becomes more evident. The percentage error of MFIV under the $skewt(5, -0.7)$ distribution increases from 2% in Table 5 to 11% in Table 8 because the truncation error and extrapolation error increases. However, percentage error of ETIV estimates remain relatively constant around 3% across different specifications.

Table 9 shows the coverage rate of ET confidence interval under 4 distributions. We present results for true volatility equal to 0.2 in the upper panel and 0.4 in the lower panel. To be more specific, we simulate 10000 states from the true distribution for 100 times, construct the confidence interval for the estimated implied volatility and see how many time the true volatility falls in the interval. We note that the higher moments, especially kurtosis of the simulated sample is far from true value in many simulations when the true distribution exhibit heavy tail. The reason is that sample kurtosis is very sensitive to the extreme observations and when the extreme observations are not realized in simulation sample, sample kurtosis is downward biased. To alleviate the bias, we only consider the simulations with kurtosis higher than 80% of the true kurtosis. From table 9, we find that the coverage rates of the confidence interval are close to the given confidence levels under the four distributions.

We also compare the performance of implied skewness and kurtosis calculated by model-free and maximum entropy method in table 10 and table 11. The formulas to calculate model-free implied skewness (MFIS) and kurtosis (MFIK) are provided in the appendix. From table 10 and 11, we see that when the underlying distribution is normal, MFIS and MFIK produce the correct estimates for risk neutral skewness and kurtosis, since the model-free moments are derived based on diffusion assumption. In this case, ETIS and ETIK are also close to the true values of skewness and kurtosis. When the underlying distribution becomes heavy tailed and negatively skewed, both of the two methods underestimate the skewness and kurtosis. In almost all cases when the underlying distribution deviates from normal, entropy method produces slightly more accurate estimates for skewness and kurtosis. It can be difficult to get accurate estimates for higher order moment because they are sensitive to tail observations and we do not give far-out-of-the-money call and put option prices. In the real-life option market, the liquidity of far-out-of-money options is also questionable except during the crisis period when the investors feel strong need to hedge the potential risks. To analyze how to further improve the estimation accuracy of higher order risk neutral moments is beyond the range of the current paper. This is also the reason we only provide confidence interval for ETIV. Confidence interval for ETIS and ETIK can be constructed when we are able to estimate higher moments with higher accuracy.

In the procedure of estimating the option implied moments by maximum entropy method, risk neutral probabilities $\hat{p}_i$, $i = 1, ..., n$, serve as an intermediate product, which allows us to compare the estimated density and the original distribution. We compare the histograms of simulated distribution (blue bars) and the distribution produced by maximum entropy method (red lines) in Figure 1. The estimated risk neutral densities in these figures are estimated from 14 options with one year maturity. Option prices with different moneyness as moment conditions essentially provide information on different parts of the distribution. For instance, out-of-the-money call option ($K_c/S = 1.15$) and out-of-money put option constraints ($K_c/S = 0.85$) restrict the right tail and the left tail of the risk neutral distribution. The figures show that risk neutral density estimated by maximum entropy method matches with simulated data both in the normal and the two skew-t cases quite well.

To conclude, we compare different method in estimating option implied risk measures in this section. Results show that maximum entropy method provides more accurate estimates of option implied volatility than Black-Scholes model and model-free method. The implied skewness and kurtosis estimated by maximum entropy also perform slightly better than that calculated by model-free method.

4 Empirical Analysis on S&P500 index option

In this section, we conduct our empirical analysis using the S&P500 index option traded in Chicago Board Options Exchange (CBOE). We estimate implied volatility, implied skewness and implied kurtosis using maximum entropy method and model-free method. We also investigate the predictive power of ETIV, BSIV, MFIV and VIX on realized volatility of S&P500 index returns in the next month.

4.1 Data

Our sample period covers from January 1996 to August 2013. We get the S&P500 index price data from The Center for Research in Security Prices (CRSP) database. We obtain S&P500

index options data From Ivy DB database of OptionMetrics. Continuously-compounded zero-coupon interest rates are also obtained from OptionMetrics as a proxy for the risk free rate. From Chicago Board Options Exchange (CBOE), we get daily levels of the newly calculated VIX index and match with trading days which have option prices with one month expiration. Although the CBOE changed the methodology for calculating the VIX in September 2003, they have backdated the new index using the historical option prices.

In this section, our analysis is conducted based on call and put option prices on S&P500 index with 30 days expiration. We choose one month maturity because the options with one month to expire are more actively traded than other maturities and the simulation study in section 2 shows that the estimation error is smaller for options with shorter time to maturity. From 1996 to 2006, only one date is available in a month with traded option data with 30 days to expire. After that, there are several dates in one month. Since the purpose of this study is to predict the volatility of S&P500 index return in the next month, we select one date in each month from 2006 to 2013, with 210 dates in total. Midpoints of the bid-ask spread are used to calculate the option implied risk measures instead of trade prices. Jackwerth (2000) demonstrates that measurement of risk neutral distribution is not sensitive to the existence of spreads.

In Table 12 and Table 13. we present the descriptive statistics of S&P500 index call options and put options with moneyness from 0.85 to 1.15. We apply several filters to select the options. First, option quotes less than 3/8 are excluded from the sample. These prices may not reflect true option value due to proximity to tick size. Second, options with zero open interest are excluded from sample due to liquidity reason. Third, following Ait-Sahalia and Lo (1998) and Bakshi and Madan (2003), we exclude in-the-money options. From the tables, we can see that for call options, the number of options and trading volume decrease with moneyness, whereas the pattern is not obvious for put options.

In the empirical study, we also consider estimation of option implied risk measures from different number of available options. To be comparable with the simulation, we first consider case A when we select call options with moneyness close to: 1.15, 1.125, 1.1, 1.075, 1.05, 1.025, 1, and put options with moneyness closest to 0.85, 0.875, 0.9, 0.925, 0.95, 0.975, 1. Then we calculate model-free and entropy-based implied moments based on the available option prices.

To make full use of the model-free method, we consider case B when we use all remaining option prices after applying the filters to calculate model-free implied moments (MFIV, MFIS, MFIK). However, when a large number of constraints are included in the entropy approach, strong correlation between moment conditions may cause the Jacobian matrix for calculating Lagrange multipliers to be ill conditioned (Buchen and Kelly (1996)). To solve the problem, we select options with strike prices not too close to each other: moneyness of call options closest to 1.5, 1.4, 1.3, 1.2, 1.175, 1.15, 1.125, 1.1, 1.075, 1.05, 1.025, 1, and moneyness of put options closest to 0.5, 0.6, 0.7, 0.8, 0.825, 0.85, 0.875, 0.9, 0.925, 0.95, 0.975, 1.

For each selected trading day, we estimate the model-free and entropy-based implied moments. For BSIV, we calculate the mean of the Black-Scholes implied volatility using all available option prices. While option implied volatilities represent ex-ante volatility forecast, we also calculate the ex-post return volatility RVD over each option's life. Following Christensen and Prabhala (1998), Christensen, Hansen and Probhala (2001) and Jiang and Tian (2005), we use monthly nonoverlapping samples to avoid the telescoping overlap problem which may render the diagnostic statistics in the regression analysis invalid. The RVD is computed as the sample standard deviation of the daily index returns over the remaining life of the option:

$$\sigma_t = \sqrt{\frac{1}{\tau_t}\sum_{k=1}^{\tau_t}(r_{t,k} - \bar{r}_t)^2}, \tag{24}$$

where τ_t is the number of the days to expiration, $\bar{r}_t = \frac{1}{\tau_t}\sum_{k=1}^{\tau_t} r_{t,k}$, and $r_{t,k}$ is the log index return on day k of month t. All of the volatility measures are expressed in annual terms to facilitate interpretation. Finally, to analyze the predictability of variance risk premium on future stock return, we calculate S&P500 index monthly return and match with the option data.

4.2 The information content of entropy-based implied volatility (ETIV)

We show our empirical findings in this section. We first review the basic statistical properties of the various volatility measures and then investigate their relative performance as predictors of the subsequent realized volatility of the underlying S&P500 index. We also analyze the predictability of different measure of variance risk premium on future index return.

Table 14 reports descriptive statistics of five measures of volatility: RVD, VIX, BSIV, MFIV and ETIV, two measures of implied skewness: MFIS and ETIS, and two measures of implied kurtosis: MFIK and ETIK calculated from 7 pairs of options. Table 15 shows the correlation matrix of these measures. We first note that the mean of the four implied volatility measure, VIX, BSIV, MFIV and ETIV, are compatible with each other from 20.9% to 21.9%. All of them exceed the mean of realized volatility measure RND by about 24%, which shows evidence of volatility risk premium. Second, the four implied volatility measures are highly correlated with each other, with correlation coefficients above 0.99. Compared to the correlation with BSIV, VIX is more correlated with MFIV and ETIV, since the construction manner of the three shares the same feature which does not depend on any parametric assumptions. Third, the two measures of implied skewness and kurtosis are correlated with coefficients 0.75 and 0.48, but the absolute level of ETIS and ETIK are higher than that of MFIS and MFIK by 43.7% and 33.4%, which confirms the results in Section 2 that model-free method tends to underestimate the absolute level of risk neutral skewness and kurtosis. From Figure 5, we can see the pattern more clearly: when implied skewness becomes lower around 1999, 2002 and 2011, the underestimation of MFIS and MFIK becomes more serious. Fourth, the implied kurtosis are negatively correlated with various volatility measures and skewness measures.

4.2.1 Forecasting Stock Market Volatility

Prior research has extensively analyzed the information content of the BSIV on the future realized volatility. While early studies produce mixed results, recent studies seem to agree the informational superiority of BSIV compared to historical volatility. Since MFIV is considered to be a better estimate of risk neutral volatility than BSIV, Jiang and Tian (2005) investigate the information content of MFIV and find it subsumes all information contained in BSIV. Although we claim that ETIV is a more accurate estimator of risk neutral volatility when the underlying distribution possesses heavy tail and skewness, it does not necessarily infer that ETIV has higher predictive power than BSIV or MFIV. If most investors form their expectations of future volatility based on simple Black-Scholes model, BSIV can be a better forecast of future volatility. Since the forecast ability of ETIV is not clear, one aim of this paper is to assess the predictive power of ETIV, compared to alternative risk neutral volatility estimators.

To nest previous research within our framework, we compare four competing volatility forecasts: BSIV, MFIV, VIX and ETIV. To explore the predictive ability of different candidate volatility measures, we first include each of them separately within an in-sample forecast regression. Denote the ex-post realized volatility for month $t+1$ as RV_{t+1} and the ith volatility predictor in a set of different predictors $I = RV_t, BSIV_t, MFIV_t, ETIV_t, VIX_t$ as $x_{i,t}$, the regressions take the form:

$$RV_{t+1} = \alpha_i + \beta_i x_{i,t} + \epsilon_{i,t+1}, \tag{25}$$

where unbiased forecasts are subject to the constraints that $\alpha_i = 0$ and $\beta_i = 1$. In addition, the regression coefficient of determination R^2 captures the proportion of total variation in the ex-post realized volatility explained by the predictors. We also employ encompassing regressions to investigate whether one predictor candidate subsumes the information in another. The regression is specified as follows:

$$RV_{t+1} = \alpha + \beta_{RV} RV_t + \beta_{BS} BSIV_t + \beta_{MF} MFIV_t + \beta_{ET} ETIV_t + \epsilon_t, \tag{26}$$

where the lagged realized volatility RV_t is our proxy for historical volatility. Following Christensen and Prabhala (1998), we adopt the realized volatility over 30 calendar days proceeding the current observation dates as the lagged realized volatility.

Table 16 and 18 summarizes the volatility regression results from both univariate and encompassing regressions for 7 pairs of options and all options in case A and B. From in-sample estimation results of univariate regressions in table 16, we note that BSIV forecasts future volatility slightly better than both MFIV and ETIV, when we only use 7 pairs of options to calculate them. In addition, MFIV and ETIV do not subsume all information in historical volatility (RVD) or BSIV. However, when we use options with strike prices span more portion of the distribution in case B, we can see some notable difference exist in the univariate regressions across model specifications and volatility measures. For instance, the adjusted R^2 is the highest for ETIV regression while it is the lowest for lagged realized volatility regression. The evidence suggests that, among all the volatility measures, ETIV explains the most variation in the future monthly volatility. It is also worth noting that even MFIV uses more options as inputs, the adjusted R^2 of ETIV regressions is still slightly higher than MFIV, while BSIV contains the least information among all the implied volatility measures. The decreased R^2 of BSIV univariate regression from table 16 to table 18 also indicates that, BSIV is unable to integrate the information in large number of cross-sectional option prices. The higher adjusted R^2 of ETIV regression may suggest that the maximum entropy method can better utilize the information contained in option prices.

The encompassing regressions in table 18 provide additional insights. First, both MFIV and ETIV subsume the information in lagged realized volatility, while BSIV does not. If the BSIV subsumes all the information in lagged realized volatility, we should expect the coefficient of lagged realized volatility to be statistically insignificant when these two measures are included in the regression. As shown in table 18, the coefficient of lagged realized volatility is not significant at 5% level in the two specifications. This finding is consistent with previous studies (e.g. Christensen and Prabhala (1998) and Jiang and Tian (2005)). Second, MFIV and ETIV subsume all information in BSIV. When ETIV or MFIV are combined with BSIV in the regression, the coefficient of BSIV is not statistically significant at 5% and the inclusion of BSIV does not improve adjusted R^2. Third, ETIV explains more variations in future realized volatility than MFIV. The highest in-sample fit obtained when ETIV is included in the predictive regressions. When the three different volatility measures are all included in the last regression, only the coefficient of ETIV is significantly different from zero. Although the correlation between the three measures distort the reliability of t statistics, the opposite signs of coefficients of MFIV and ETIV indicate the dominant role of ETIV in explaining variations of future volatility.

We then turn to the out-of-sample evidence reported in right columns of table 16 and table 18. The out-of-sample results in table 16 and 18 do not differ much, while table 18 provides more evident results on the superiority of ETIV in out-of-sample performance. We use rolling window of the previous 100 observations as estimation sample and the remaining ones as prediction sample. The overall measure of forecast performance is the percentage

RMSE. If we denote $\hat{y}_t$ as a forecast for y_t, it is formally defined as,

$$RMSE = \frac{\sqrt{E[(\hat{y} - y)^2]}}{E[y^2]}/100$$

The column labeled "All days" covers the full out-of-sample period. The relative ranking is consistent with the in-sample results. From the univariate regressions in table 18, we find that ETIV continues to provide the best performance among other volatility measures. Forecast precision deteriorate monotonically as we move to MFIV, VIX, BSIV and RVD. Moreover, the RMSEs of ETIV (0.179 and 0.145) in a univariate regression in the two tables are smaller than other combinations of volatility measure, which suggest the forecast superiority of ETIV. The last three columns in table 10 and table 11 report out-of-sample RMSE of three subsamples. We divide the monthly forecast of future volatility into three subsamples by sorting the corresponding BSIV measure in ascending order. Hence, RMSE results for monthly forecasts with the lowest third BSIV are reported in the "Low" column, results for the next third are in the "Medium" column and for the last third with the highest BSIV in the "High" column. First, from the univariate regressions results, we note that ETIV performs better than other volatility measure in the high volatility regime, while all measures perform comparably in the low volatility regime. Second, the encompassing regressions largely confirm the observations drawn from the univariate out-of-sample predictive regressions. When RVD is combined with BSIV, MSIV and MFIV, the rank of out-of-sample performance is the same as in univariate regressions. In addition, once ETIV is included in the regression, the addition of either RVD, BSIV or MFIV does not improve the out-of-sample performance. It is notable that although the combination of RVD, MFIV and ETIV provides the best in-sample fit, it has much worse out-of-sample predictive power than other combinations. Compare table 16 and table 18, we find that the forecasting performance of MFIV and ETIV are improved using more option prices, both in-sample and out-of-sample. The superiority of ETIV compared to MFIV and BSIV arises when we try to explore all information in the available index option data.

4.2.2 Forecasting Stock Market Returns

At last, we investigate the relation between variance risk premium and future market return. The theoretical model in Bollerslev, Tauchen and Zhou (2009) suggest that variance risk premium (VRP) may serve as a useful predictor for the future returns. VRP is defined as the difference between current return variation and the markets risk-neutral expectation of future return variation. In this paper, we intend to compare the performance of VRP using different measure of risk neutral variance. To explore the predictability of different VRP measures, we use univariate regression to examine the in-sample and out-of-sample performance. Denote the ex-post return for month $t+1$ as R_{t+1} and the ith VRP measure in a set of different predictors $I = VRP_{BS,t}, VRP_{MF,t}, VRP_{ET,t}$ as $x_{i,t}$, the regressions take the form:

$$R_{t+1} = \alpha_i + \beta_i x_{i,t} + \epsilon_{i,t+1}, \tag{27}$$

where $VRP_{BS,t}$ is defined as the difference between the Black-Scholes implied variance$BSIV_t^2$ and realized variance in the past month RVD_t^2. $VRP_{MF,t}$ and $VRP_{ET,t}$ are calculated based on $MFIV_t$ and $ETIV_t$.

Table 17 and 19 report regression result of predicting future monthly market returns. The VRP measures are calculated based on 7 pairs of options and all available options respectively. In all regressions, the t-statistics for testing the estimated slope coefficient associated with the VRP measures greater than zero exceeds the one-sided 2.5% significance level. Furthermore, both tables show that the VRP_{ET} explains more variations in future monthly market return than VRP_{BS} and VRP_{MF} and performs better in out-of-sample setup. The out-of-sample RMSE is the lowest for VRP_{ET} in the high volatility regime, which also support our results in simulation that ETIV is a more accurate estimator of risk neutral volatility when the underlying distribution has higher volatility. Compare table 17 with table 19, we find that increasing the number of options to calculate VRP measures enhances the predictability of VRP. The improvement of VRP_{BS} and VRP_{MF} is larger than VRP_{ET}, whereas the performance of VRP_{ET} almost stays unchanged using different set of options. In addition, the adjusted R^2 reported in table 17 and 19 for VRP_{ET} regression is 7.4%, which is much larger than 1.07% reported in Bollerslev, Tauchen and Zhou (2009). We explain the improvement as a result of ETIV as a more accurate estimator of risk-neutral expectation of future return variation.

5 Conclusion

This paper provides the first comprehensive investigation on option implied moments estimated by principle of maximum entropy. The method estimates the risk neutral distribution of an asset, given a set of option prices at different strikes. Implied volatility (ETIV), implied skewness (ETIS) and implied kurtosis (ETIK), can then be calculated based on the estimated risk neutral distribution. Compared to parametric methods such as Black Scholes (BS) model, the proposed method does not depend on any parametric assumptions. Compared to model-free implied volatility, such as that in Bakshi and Madan (2003), this method does not require options with exercise prices spanning the full range of possible values for the underlying asset at expiry. Instead, the entropy method combines the advantages in model-free and parametric methods: on one hand, it can aggregate information in options with different strikes and produce accurate estimates using only limit number of options; on the other hand, constructing confidence interval for option implied moments is also possible since a nonparametric analog of likelihood ratio statistics follows chi-square distribution under certain assumptions.

We first investigate the performance of maximum entropy method when the risk neutral distribution of the underlying log return follows normal, student t, and skewed student t distributions. Given certain risk neutral distributions, put and call options are calculated by numerical integration, and implied moments are then backed out by Black Scholes model, model-free method and ET method. We find that ETIV has less estimation error than BSIV and MFIV when the underlying distribution shows heavy tail and non-zero skewness. With less number of available options or under higher true volatility level, the accuracy of ETIV remains robust while the percentage error of MFIV becomes larger. The implied skewness and kurtosis estimated by ET (ETIS and ETIK) are also slightly more accurate than their counterparts calculated by model-free methods (MFIS and MFIK). In addition, a confidence interval can be constructed for ETIV, which coverage is close to the correct confidence level under different distributions. The complete distributions estimated by ET method also match with the given distribution to a high degree of accuracy.

Using S&P500 index options, we empirically test the information content of ETIV on future monthly realized volatility and return. Our in-sample regression results using all available options support the hypothesis that ETIV subsumes all information in BSIV and lagged realized volatility and has higher predictive power on future monthly volatility than MFIV. In the out-of-sample analysis, ETIV continues to provide superior forecasts and performs the best in high volatility regime. When forecasting future monthly return, entropy-based variance risk premium continues to explain more variations in future monthly return and performs better in the out-of-sample setup, using different number of option prices. To conclude, empirical evidence supports our results in simulation that ETIV is a more accurate estimator of risk neutral expectation of future return variation. Further improvement of the estimation accuracy of option implied skewness and kurtosis should be an important next step of further research.

6 Appendix

6.1 Density function of log Skewed Student's t distribution

Hansen (1994) suggests a Skewed Student's t distribution to allow for skewness in Student's t distribution. The density function of Skewed Student's t distribution is:

$$f(x) = \begin{cases} bc(1 + \frac{1}{\eta-2}(\frac{bx+a}{1-\lambda})^2)^{-(1+\eta)/2} & \text{if } x < -a/b, \\ bc(1 + \frac{1}{\eta-2}(\frac{bx+a}{1+\lambda})^2)^{-(1+\eta)/2} & \text{if } x \geq -a/b, \end{cases}$$

where $2 < \eta < \infty$, and $-1 < \lambda < 1$. The constants a, b and c are given by:

$$a = 4\lambda c(\frac{\eta-2}{\eta-1}), \;\; b^2 = 1 + 3\lambda^2 - a^2, \;\; \text{and} \;\; c = \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta-2)\Gamma(\eta/2)}}.$$

In the second and the third case, we model the continuous compounded return X as skewed Student's t distribution. Assume the density function of X is $f(x)$, then the density function of the discrete return $Y = e^X$ can be expressed as:

$$g(y) = f(\ln y)\frac{1}{y} = \begin{cases} \frac{b}{y}c(1 + \frac{1}{\eta-2}(\frac{b\ln y+a}{1-\lambda})^2)^{-(1+\eta)/2} & \text{if } \ln y < -a/b, \\ \frac{b}{y}c(1 + \frac{1}{\eta-2}(\frac{b\ln y+a}{1+\lambda})^2)^{-(1+\eta)/2} & \text{if } \ln y \geq -a/b. \end{cases}$$

6.2 Calculation of Model-free implied moments

The calculation of model-free option implied moments in this paper follows Bakshi, Kapadia and Madan (2003). Let the t-period continuous compounded return be given by: $R_t = \ln[S_t] - \ln[S_0]$. The fair value of the volatility contract, the cubic contract and the quartic contract at time 0 are:

$$V(0,t) = E^Q[e^{-rt}R_t^2], \;\; W(0,t) = E^Q[e^{-rt}R_t^3], \;\; \text{and} \;\; X(0,t) = E^Q[e^{-rt}R_t^4]$$

To simplify the notations, we ignore the time period information in the parenthesis in the following equations, for instance$V = V(0,t)$. Under the risk neutral measure, the following skewness and kurtosis contract prices can be recovered by the out-of-the-money European call and put option prices. The t-period risk neutral return skewness, $SKEW$ is given by:

$$SKEW = \frac{e^{rt}W - 3\mu e^{rt}V + 2\mu^3}{(e^r tV - \mu^2)^{3/2}}.$$

The t-period risk neutral kurtosis, $KURT$ is:

$$KURT = \frac{e^{rt}X - 4\mu e^{rt}W + 6e^{rt}\mu^2 V - 3\mu^4}{e^{rt}V - \mu},$$

where μ, V, W and X can be replicated by the option prices:

$$\mu = E^Q(\ln[\frac{S_t}{S_0}]) = e^{rt}(1 - e^{-rt} - \frac{1}{2}V - \frac{1}{6}W - \frac{1}{24}X),$$

$$V = \int_S^\infty \frac{2(1 - \ln[\frac{K}{S_0}])}{K^2} C(K,t)dK + \int_0^S \frac{2(1 + \ln[\frac{S_0}{K}])}{K^2} P(K,t)dK,$$

$$W = \int_S^\infty \frac{6\ln[\frac{K}{S}] - 3(\ln[\frac{K}{S_0}])^2}{K^2} C(K,t)dK - \int_0^S \frac{6\ln[\frac{K}{S}] + 3(\ln[\frac{S_0}{K}])^2}{K^2} P(K,t)dK,$$

$$X = \int_S^\infty \frac{12(\ln[\frac{K}{S}])^2 - 4(\ln[\frac{K}{S_0}])^3}{K^2} C(K,t)dK - \int_0^S \frac{12(\ln[\frac{K}{S})^2] + 4(\ln[\frac{S_0}{K}])^3}{K^2} P(K,t)dK.$$

Table 1: 7 pairs of strike prices (K/S_0) for different maturities

		1	2	3	4	5	6	7
$T = 1/12$	call	1.15	1.125	1.1	1.075	1.05	1.025	1
	put	0.85	0.875	0.9	0.925	0.95	0.975	1
$T = 1/6$	call	1.2	1.15	1.125	1.1	1.075	1.025	1
	put	0.83	0.85	0.875	0.9	0.925	0.975	1
$T = 1/4$	call	1.27	1.2	1.15	1.1	1.075	1.05	1
	put	0.8	0.83	0.85	0.9	0.925	0.95	1
$T = 1/2$	call	1.4	1.3	1.2	1.15	1.1	1.05	1
	put	0.75	0.8	0.83	0.85	0.9	0.95	1
$T = 1$	call	1.65	1.5	1.35	1.2	1.1	1.05	1
	put	0.65	0.7	0.75	0.8	0.9	0.95	1

Table 2: 5 pairs of strike prices (K/S_0) for maturities from $T = 1/12$ to $T = 1$

	1	2	3
call	1.05	1.025	1
put	0.95	0.975	1

The upper table reports the 7 pairs of strike prices (K/S_0) of call and put options we use in the calculation of BS, MF and ET implied volatility. The range of strike prices is adjusted by different maturities. The upper and lower bounds of the strike prices are determined by $exp(\Phi^{-1}(0.01)\sigma_t + \mu_t)$,$exp(\Phi^{-1}(0.99)\sigma_t + \mu_t)$, where $\Phi^{-1}(x)$ is the inverse cumulative distribution function of the standard normal distribution. The lower table reports 3 pairs of strike prices (K/S_0) of call and put options which do not change according to different maturities.

Table 3: Call option prices under different risk neutral distributions

K_c/S	Distribution	Time to Expiration (Years)				
		1/12	1/6	1/4	1/2	1
1	lognormal	0.020	0.053	0.045	0.080	0.360
	t	0.078	0.153	0.169	0.290	0.767
	skewt(5,-0.3)	0.020	0.042	0.044	0.076	0.244
	skewt(5,-0.7)	0.000	0.001	0.001	0.001	0.005
2	lognormal	0.057	0.198	0.200	0.306	0.785
	t	0.125	0.298	0.336	0.532	1.147
	skewt(5,-0.3)	0.038	0.110	0.116	0.183	0.453
	skewt(5,-0.7)	0.001	0.004	0.003	0.005	0.018
3	lognormal	0.148	0.360	0.514	1.023	1.640
	t	0.210	0.432	0.599	1.120	1.833
	skewt(5,-0.3)	0.080	0.191	0.275	0.571	0.951
	skewt(5,-0.7)	0.003	0.012	0.021	0.071	0.126
4	lognormal	0.349	0.626	1.191	1.762	3.247
	t	0.373	0.644	1.139	1.710	3.130
	skewt(5,-0.3)	0.188	0.350	0.736	1.101	2.212
	skewt(5,-0.7)	0.022	0.060	0.293	0.426	1.166
5	lognormal	0.744	1.042	1.742	2.906	6.040
	t	0.691	0.978	1.597	2.672	5.589
	skewt(5,-0.3)	0.474	0.659	1.217	2.144	5.118
	skewt(5,-0.7)	0.237	0.309	0.817	1.581	4.480
6	lognormal	1.435	2.524	2.478	4.582	8.021
	t	1.292	2.293	2.245	4.200	7.484
	skewt(5,-0.3)	1.146	2.149	1.960	3.946	7.406
	skewt(5,-0.7)	1.002	1.994	1.670	3.625	7.011
7	lognormal	2.512	3.675	4.615	6.889	10.451
	t	2.333	3.424	4.312	6.478	9.925
	skewt(5,-0.3)	2.336	3.437	4.340	6.576	10.255
	skewt(5,-0.7)	2.310	3.396	4.284	6.478	10.066

This table reports call option prices with different moneyness and maturity under different risk neutral distributions. Risk neutral distributions of the continuously compounded stock returns are simulated from normal and two skew-t distributions. K_c is the exercise price of the call option and S is the current price of the stock.

Table 4: Put option prices under different risk neutral distributions

K_p/S	Distribution	Time to Expiration (Years)				
		1/12	1/6	1/4	1/2	1
1	lognormal	0.003	0.021	0.028	0.057	0.042
	t	0.029	0.072	0.088	0.138	0.127
	skewt(5,-0.3)	0.062	0.139	0.168	0.255	0.248
	skewt(5,-0.7)	0.093	0.196	0.235	0.347	0.344
2	lognormal	0.015	0.049	0.079	0.199	0.126
	t	0.054	0.109	0.149	0.280	0.227
	skewt(5,-0.3)	0.105	0.195	0.260	0.457	0.403
	skewt(5,-0.7)	0.149	0.264	0.345	0.580	0.529
3	lognormal	0.061	0.127	0.148	0.377	0.317
	t	0.107	0.185	0.215	0.430	0.401
	skewt(5,-0.3)	0.184	0.300	0.350	0.648	0.647
	skewt(5,-0.7)	0.242	0.385	0.447	0.788	0.803
4	lognormal	0.193	0.287	0.552	0.554	0.687
	t	0.222	0.318	0.543	0.571	0.702
	skewt(5,-0.3)	0.329	0.466	0.740	0.816	1.024
	skewt(5,-0.7)	0.402	0.564	0.859	0.963	1.203
5	lognormal	0.504	0.584	0.951	1.276	2.310
	t	0.469	0.553	0.862	1.148	2.020
	skewt(5,-0.3)	0.598	0.726	1.074	1.437	2.431
	skewt(5,-0.7)	0.675	0.828	1.189	1.582	2.584
6	lognormal	1.106	1.816	1.534	2.527	3.713
	t	0.979	1.610	1.352	2.218	3.253
	skewt(5,-0.3)	1.086	1.733	1.549	2.470	3.624
	skewt(5,-0.7)	1.137	1.780	1.641	2.556	3.694
7	lognormal	2.096	2.845	3.373	4.420	5.574
	t	1.917	2.593	3.067	3.998	5.005
	skewt(5,-0.3)	1.922	2.610	3.100	4.093	5.259
	skewt(5,-0.7)	1.899	2.577	3.059	4.036	5.184

This table reportscall option prices with different moneyness and maturity under different risk neutral distributions. Risk neutral distributions of the continuously compounded stock returns are simulated from normal and two skew-t distributions. K_p is the exercise price of the put option and S is the current price of the stock.

Table 5: Implied volatilities estimated from 7 pairs of options, $\sigma = 0.2$

	Method	1/12	1/6	1/4	1/2	1
Normal	BS	0.200	0.200	0.200	0.200	0.200
	MF	0.200	0.200	0.200	0.200	0.200
	ET	0.200	0.200	0.201	0.202	0.204
student t	BS	0.211	0.199	0.195	0.190	0.188
	MF	0.198	0.196	0.194	0.192	0.190
	ET	0.199	0.198	0.197	0.197	0.197
skewt(5,-0.3)	BS	0.206	0.196	0.192	0.190	0.191
	MF	0.197	0.195	0.194	0.192	0.193
	ET	0.198	0.197	0.197	0.198	0.200
skewt(5,-0.7)	BS	0.195	0.186	0.184	0.184	0.188
	MF	0.196	0.193	0.191	0.189	0.190
	ET	0.197	0.196	0.195	0.196	0.193

This table reports the estimated implied volatility calculated from 7 pairs of option prices by Black Sholes formula (BS), model-free method (MF) and maximum entropy method (ET) under different risk neutral distributions. The true volatility is 0.2. In the first row, 1/12 to 1 means options maturities from one month to a year. In the first column, 'skewt(5,-0.3)' means Skewed Student t distribution with degree of freedom 5 and skewness parameter -0.3; 'skewt(5,-0.7)' means Skewed Student t distribution with degree of freedom 5 and skewness parameter -0.7.

Table 6: Implied volatilities estimated from 3 pairs of options, $\sigma = 0.2$

	Method	1/12	1/6	1/4	1/2	1
Normal	BS	0.200	0.200	0.200	0.200	0.200
	MF	0.200	0.200	0.200	0.200	0.200
	ET	0.202	0.201	0.202	0.201	0.202
student t	BS	0.192	0.193	0.192	0.193	0.192
	MF	0.195	0.194	0.193	0.195	0.192
	ET	0.196	0.196	0.196	0.197	0.197
skewt(5,-0.3)	BS	0.192	0.190	0.188	0.191	0.193
	MF	0.197	0.193	0.191	0.195	0.193
	ET	0.196	0.196	0.196	0.197	0.199
skewt(5,-0.7)	BS	0.187	0.180	0.177	0.180	0.183
	MF	0.196	0.189	0.186	0.192	0.187
	ET	0.193	0.193	0.193	0.194	0.196

This table reports the estimated implied volatility calculated from 3 pairs of option prices by Black Sholes formula (BS), model-free method (MF) and maximum entropy method (ET) under different risk neutral distributions. The true volatility is 0.2. In the first row, numbers from 1/12 to 1 represent options maturities from one month to a year. In the first column, 'skewt(5,-0.3)' means Skewed Student t distribution with degree of freedom 5 and skewness parameter -0.3; 'skewt(5,-0.7)' means Skewed Student t distribution with degree of freedom 5 and skewness parameter -0.7.

Table 7: Implied volatilities estimated from 7 pairs of options, $\sigma = 0.4$

	Method	1/12	1/6	1/4	1/2	1
Normal	BS	0.400	0.400	0.400	0.400	0.400
	MF	0.400	0.400	0.400	0.400	0.400
	ET	0.402	0.403	0.404	0.406	0.407
student t	BS	0.385	0.383	0.384	0.386	0.392
	MF	0.387	0.385	0.386	0.387	0.390
	ET	0.393	0.394	0.395	0.397	0.401
skewt(5,-0.3)	BS	0.374	0.369	0.368	0.365	0.366
	MF	0.383	0.379	0.379	0.378	0.382
	ET	0.391	0.390	0.391	0.394	0.399
skewt(5,-0.7)	BS	0.350	0.345	0.341	0.334	0.331
	MF	0.375	0.369	0.367	0.364	0.366
	ET	0.384	0.383	0.384	0.385	0.389

This table reports the estimated implied volatility calculated from 7 pairs of option prices by Black Sholes formula (BS), model-free method (MF) and maximum entropy method (ET) under different risk neutral distributions. The true volatility is 0.4. In the first row, numbers from 1/12 to 1 represent options maturities from one month to a year. In the first column, 'skewt(5,-0.3)' means Skewed Student t distribution with degree of freedom 5 and skewness parameter -0.3; 'skewt(5,-0.7)' means Skewed Student t distribution with degree of freedom 5 and skewness parameter -0.7.

Table 8: Implied volatilities estimated from 3 pairs of options, $\sigma = 0.4$

	Method	1/12	1/6	1/4	1/2	1
student t	BS	0.373	0.375	0.376	0.379	0.386
	MF	0.374	0.375	0.376	0.378	0.381
	ET	0.393	0.394	0.397	0.400	0.406
skewt(5,-0.3)	BS	0.368	0.364	0.361	0.361	0.363
	MF	0.366	0.365	0.363	0.367	0.368
	ET	0.391	0.391	0.393	0.396	0.404
skewt(5,-0.7)	BS	0.359	0.348	0.341	0.338	0.334
	MF	0.355	0.351	0.346	0.351	0.346
	ET	0.387	0.384	0.384	0.387	0.393

This table reports the estimated implied volatility calculated from 3 pairs of option prices by Black Sholes formula (BS), model-free method (MF) and maximum entropy method (ET) under different risk neutral distributions. The true volatility is 0.4. In the first row, numbers from 1/12 to 1 represent options maturities from one month to a year. In the first column, 'skewt(5,-0.3)' means Skewed Student t distribution with degree of freedom 5 and skewness parameter -0.3; 'skewt(5,-0.7)' means Skewed Student t distribution with degree of freedom 5 and skewness parameter -0.7.

Table 9: Coverage rate of ET confidence interval under different distributions

sigma		normal	t	skewt1	skewt2
0.2	95%	91.21%	91.40%	92.30%	93.10%
	90%	85.00%	85.50%	86.60%	92.30%
0.4	95%	92.39%	91.60%	93.40%	92.50%
	90%	88.78%	86.70%	87.60%	84.50%

This table reports the coverage rate of confidence interval under four distributions. The upper panel is for $\sigma = 0.2$ under 95% and 90% confidence levels and the lower panel is for $\sigma = 0.4$.

Table 10: Implied skewness estimated by model-free method and entropy method

	Method	1/12	1/6	1/4	1/2	1
Normal, skew=0	MF	0.000	0.000	0.000	0.000	0.000
	ET	0.001	0.004	0.004	-0.013	-0.066
student t, skew=0	MF	-0.021	-0.026	-0.026	-0.023	-0.013
	ET	-0.002	-0.008	-0.017	-0.048	-0.104
skewt(5,-0.3), skew= -1.23	MF	-0.896	-0.781	-0.717	-0.606	-0.477
	ET	-0.917	-0.872	-0.857	-0.848	-0.850
skewt(5,-0.7), skew=-2.06	MF	-1.479	-1.297	-1.196	-1.020	-0.799
	ET	-1.512	-1.433	-1.399	-1.360	-1.137

This table reports the estimated implied skewness calculated from 7 pairs of options by model-free method (MF) and entropy method (ET). The first column shows different risk neutral distributions for calculating option prices and their true skewness parameters. The first row represents five maturities from 1 month to 1 year.

Table 11: Implied kurtosis estimated by model-free method and entropy method

	Method	1/12	1/6	1/4	1/2	1
normal, kurt=3	MF	3.005	3.005	3.005	3.005	3.005
	ET	3.007	3.077	3.179	3.450	3.758
student t, kurt=9	MF	4.878	4.156	3.817	3.406	3.179
	ET	5.254	4.898	4.724	4.525	4.456
skewt(5,-0.3), kurt=11.88	MF	5.402	4.496	4.075	3.562	3.259
	ET	5.678	5.242	5.050	4.850	4.791
skewt(5,-0.7), kurt=17.54	MF	6.328	5.117	4.569	3.902	3.510
	ET	6.577	6.016	5.789	5.553	4.696

This table reports the estimated implied kurtosis calculated from 7 pairs of options by model-free method (MF) and entropy method (ET). The first column shows different risk neutral distributions for calculating option prices and their true kurtosis parameters. The first row represents five maturities from 1 month to 1 year.

Table 12: Descriptive statistics of S&P500 call options with 1 month expiration

K_c/S	1	1.025	1.05	1.075	1.1	1.125	1.15
Mean	24.82	11.56	5.26	3.17	2.19	2.48	1.85
Variance	84.02	58.48	35.76	21.95	15.25	19.99	13.50
Skewness	1.05	1.30	3.03	4.36	4.89	3.78	3.86
Kurtosis	4.90	5.75	17.14	27.53	30.80	17.50	17.82
Maximum	7.69	1.94	0.40	0.38	0.38	0.38	0.38
Minimum	64.85	51.15	45.20	36.80	29.30	24.50	19.30
Average volume	4763.65	3265.00	2282.02	2030.98	1371.10	1229.62	1621.00
Average open interest	14338.16	16346.00	15054.36	14399.36	10743.60	13956.85	21618.22
obs.	204	206	193	139	96	41	32

This table presents the descriptive statistics of the S&P500 call options with 1 month expiration. The first row shows different moneyness from at-of-the-money ($K_c/S = 1$) to out-of-the-money ($K_c/S = 1.15$). K_c is the exercise price of the call option and S is the current price of S&P500 index.

Table 13: Descriptive statistics of S&P500 put options with 1 month expiration

K_p/S	0.85	0.875	0.9	0.925	0.95	0.975	1
Mean	2.36	2.91	4.04	5.90	9.07	14.72	24.23
Variance	10.92	14.59	21.08	32.16	49.06	70.67	85.75
Skewness	4.22	4.00	3.45	2.82	2.25	1.55	0.94
Kurtosis	25.45	23.76	19.25	14.51	10.79	6.97	4.77
Maximum	0.38	0.38	0.45	0.53	1.33	3.65	8.25
Minimum	25.20	30.05	34.45	40.35	47.35	55.25	62.65
Average volume	1358.20	1983.32	2630.10	3182.80	2441.32	3163.30	4515.67
Average open interest	11739.98	16769.85	18331.39	21581.85	19213.53	18214.70	12526.01
obs.	163	191	199	201	201	204	206

This table presents the descriptive statistics of the S&P500 put options with 1 month expiration. The first row shows different moneyness from out-of-the-money ($K_p/S = 0.85$) to at-the-money ($K_p/S = 1$). K_p is the exercise price of the call option and S is the current price of S&P500 index.

Table 14: Descriptive statistics of different measures of implied volatility, skewness and kurtosis

	Mean	Median	Std. Dev.	Skewness	Kurtosis	Maximum	Minimum
RVD	0.172	0.145	0.101	2.739	14.158	0.784	0.055
VIX	0.217	0.200	0.093	2.460	13.122	0.809	0.102
BSIV	0.219	0.204	0.078	2.994	17.341	0.765	0.130
MFIV	0.209	0.195	0.085	2.460	13.349	0.758	0.103
MFIS	-1.035	-1.046	0.342	0.280	2.776	0.117	-1.726
MFIK	5.907	5.417	1.979	0.945	3.565	12.771	2.932
ETIV	0.210	0.194	0.089	2.257	11.343	0.745	0.097
ETIS	-1.483	-1.453	0.397	-0.588	3.855	-0.547	-2.988
ETIK	7.873	7.432	2.589	1.554	7.415	21.060	2.368

This table reports the descriptive statistics for volatility measures RVD, VIX, BSIV, MFIV and ETIV, implied skewness measures MFIS and ETIS and implied kurtosis measures MFIK and ETIK.Statistics are reported for the full sample from January 1996 to August 2013. In all tables and figures, the volatility measures are annulized and given in decimal form.

Table 15: Correlation matrix of different option implied risk measures

	RVD	VIX	BSIV	MFIV	MFIS	MFIK	ETIV	ETIS	ETIK
RVD	1.000	0.734	0.735	0.735	0.408	-0.525	0.735	0.237	-0.342
VIX	0.734	1.000	0.994	0.998	0.545	-0.700	0.998	0.192	-0.362
BSIV	0.735	0.994	1.000	0.996	0.566	-0.662	0.993	0.235	-0.381
MFIV	0.735	0.998	0.996	1.000	0.573	-0.714	0.998	0.220	-0.381
MFIS	0.408	0.545	0.566	0.573	1.000	-0.709	0.540	0.752	-0.692
MFIK	-0.525	-0.700	-0.662	-0.714	-0.709	1.000	-0.707	-0.341	0.485
ETIV	0.735	0.998	0.993	0.998	0.540	-0.707	1.000	0.183	-0.353
ETIS	0.237	0.192	0.235	0.220	0.752	-0.341	0.183	1.000	-0.913
ETIK	-0.342	-0.362	-0.381	-0.381	-0.692	0.485	-0.353	-0.913	1.000

This table reports the correlations for various measures of volatility, skewness and kurtosis. Th sample period is January 1996 to August 2013.

Table 16: Volatility Regressions (7 pairs of options)

	In-Sample Estimation					Out-of-sample RMSE			
	α	β_1	β_2	β_3	$adj.R^2$	All days	Low	Medium	High
RVD	0.052***	0.673***			0.474	0.406	0.423	0.483	0.376
	(5.219)	(13.752)							
BSIV	-0.027***	0.916***			0.542	0.391	0.249	0.485	0.370
	(-1.979)	(15.765)							
VIX	-0.002	0.800***			0.537	0.393	0.242	0.485	0.373
	(-0.145)	(15.609)							
MFIV	-0.005	0.817***			0.539	0.391	0.242	0.487	0.370
	(-0.382)	(15.649)							
ETIV	0.011	0.756***			0.536	0.391	0.240	0.484	0.371
	(0.984)	(15.580)							
RVD+BSIV	-0.016	0.160	0.735***		0.546	0.391	0.260	0.481	0.370
	(-1.055)	(1.614)	(5.828)						
RVD+MFIV	0.002	0.176*	0.639***		0.543	0.390	0.253	0.482	0.369
	(0.193)	(1.800)	(5.724)						
RVD+ETIV	0.015	0.182*	0.585***		0.542	0.390	0.253	0.479	0.370
	(1.312)	(1.859)	(5.644)						
BSIV+MFIV	-0.034	1.226	-0.278		0.540	0.406	0.267	0.486	0.390
	(-1.352)	(1.336)	(-0.338)						
BSIV+ETIV	-0.035	1.125*	-0.174		0.540	0.409	0.285	0.481	0.396
	(-1.167)	(1.665)	(-0.310)						
ETIV+MFIV	-0.005	0.814	0.003		0.536	0.400	0.261	0.486	0.382
	(-0.237)	(0.998)	(0.003)						
BSIV+MFIV+ETIV	-0.036	1.235	-0.195	-0.084	0.538	0.422	0.278	0.484	0.415
	(-1.177)	(1.337)	(-0.176)	(-0.111)					

This table presents regression results of forecasting realized volatility using different measures of volatility. The implied volatility measures are calculated from 7 pairs of options. The sample period extends from January 1996 to August 2013. All of the regressions are based on monthly nonoverlap observations. t-statistics are reported in parentheses. The dependent variable is the realized volatility in the next month defined in equation (24). For out-of-sample analysis, we split the whole sample into three subsamples by sorting the corresponding BSIV measure in ascending order. "Low", "Medium" and "High" represent different volatility regime.

Table 17: Return Regressions (7 pairs of options)

	In-Sample Estimation			Out-of-sample RMSE			
	α	β_1	$adj.R^2$	All days	Low	Medium	High
VRP_{BS}	0.001	0.348***	0.045	0.982	1.058	1.010	0.956
	(0.146)	(3.295)					
VRP_{MF}	-0.001	0.402***	0.059	0.973	1.070	1.008	0.939
	(-0.169)	(3.740)					
VRP_{ET}	-0.001	0.448***	0.075	0.965	1.073	1.010	0.922
	(-0.308)	(4.226)					

This table presents predictive regression results of variance risk premium on future monthly return, where variance risk premium is calculated based on 7 pairs of option prices.The sample period extends from January 1996 to August 2013. All of the regressions are based on monthly nonoverlap observations. t-statistics are reported in parentheses. VRP_{BS} is the variance risk premium calculated by the difference between $BSIV^2$ and realized variance in the last month RVD^2. VRP_{MF} and VRP_{ET} are variance risk premium calculated based on MFIV and ETIV.

Table 18: Volatility Regressions, all options

	In-Sample Estimation					Out-of-sample RMSE			
	α	β_1	β_2	β_3	$adj.R^2$	All days	Low	Medium	High
RVD	0.052***	0.673***	-	-	0.474	0.406	0.445	0.458	0.382
	(5.219)	(13.752)	-	-					
BSIV	-0.001	0.736***	-	-	0.530	0.391	0.313	0.472	0.371
	-(0.094)	(15.377)	-	-					
VIX	-0.002	0.800***	-	-	0.537	0.393	0.304	0.468	0.376
	-(0.145)	(15.609)	-	-					
MFIV	0.009	0.731***	-	-	0.543	0.389	0.304	0.470	0.370
	(0.762)	(15.779)	-	-					
ETIV	0.009	0.767***	-	-	0.552	0.386	0.300	0.467	0.366
	(0.764)	(16.092)	-	-					
RVD+BSIV	0.006	0.217**	0.540***	-	0.539	0.386	0.323	0.462	0.366
	(0.516)	(2.295)	(5.529)	-					
RVD+MFIV	0.012	0.158	0.589***	-	0.546	0.388	0.312	0.464	0.370
	(1.076)	(1.589)	(5.839)	-					
RVD+ETIV	0.011	0.119	0.655***	-	0.553	0.386	0.306	0.463	0.368
	(0.994)	(1.186)	(6.168)	-					
BSIV+MFIV	0.013	-0.260	0.985***	-	0.541	0.392	0.298	0.468	0.376
	(0.988)	(-0.649)	(2.501)	-					
BSIV+ETIV	0.017	-0.406	1.178***	-	0.553	0.389	0.294	0.466	0.373
	(1.281)	(-1.222)	(3.470)	-					
MFIV+ETIV	0.012	-1.778**	2.613***		0.560	0.409	0.297	0.466	0.402
	(1.101)	(-2.152)	(3.042)						
BSIV+MFIV+ETIV	0.012***	0.001	-1.779	2.613***	0.558	0.414	0.292	0.464	0.410
	(7.005)	(0.629)	(-0.327)	(4.141)					

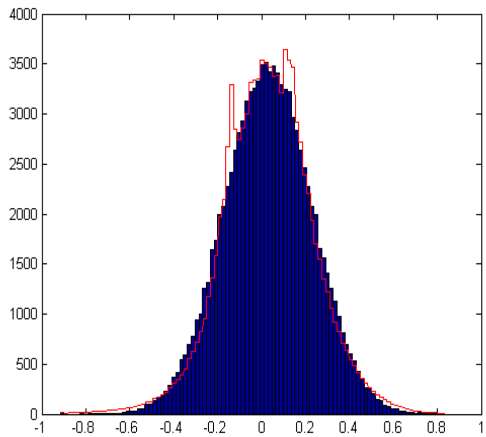
This table presents regression results of forecasting realized volatility using different measures of volatility. The implied volatility measures are calculated from all options. The sample period extends from January 1996 to August 2013. All of the regressions are based on monthly nonoverlap observations. t-statistics are reported in parentheses. The dependent variable is the realized volatility in the next month defined in equation (24). For out-of-sample analysis, we split the whole sample into three subsamples by sorting the corresponding BSIV measure in ascending order. “Low”, “Medium” and “High” represent different volatility regime.

Table 19: Return Regressions (all options)

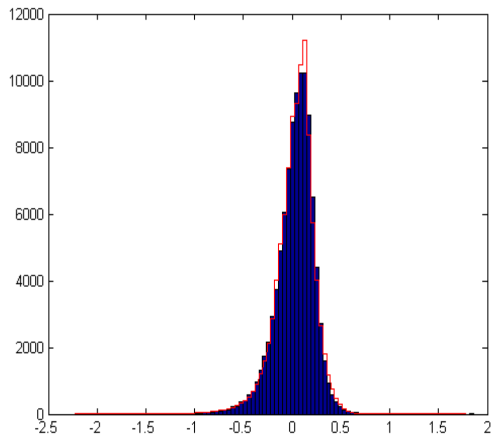
	In-Sample Estimation			Out-of-sample RMSE			
	α	β_1	$adj.R^2$	All days	Low	Medium	High
VRP_{BS}	-0.005	0.416***	0.067	0.974	1.070	1.005	0.937
	(-1.196)	(4.002)					
VRP_{MF}	-0.003	0.435***	0.069	0.969	1.063	1.003	0.929
	(-0.827)	(4.059)					
VRP_{ET}	-0.002	0.480***	0.074	0.962	1.057	1.003	0.918
	(-0.423)	(4.219)					

This table presents predictive regression results of variance risk premium on future monthly return, where variance risk premium is calculated based on all option prices. The sample period extends from January 1996 to August 2013. All of the regressions are based on monthly nonoverlap observations. t-statistics are reported in parentheses. VRP_{BS} is the variance risk premium calculated by the difference between $BSIV^2$ and realized variance in the last month RVD^2. VRP_{MF} and VRP_{ET} are variance risk premium calculated based on MFIV and ETIV.

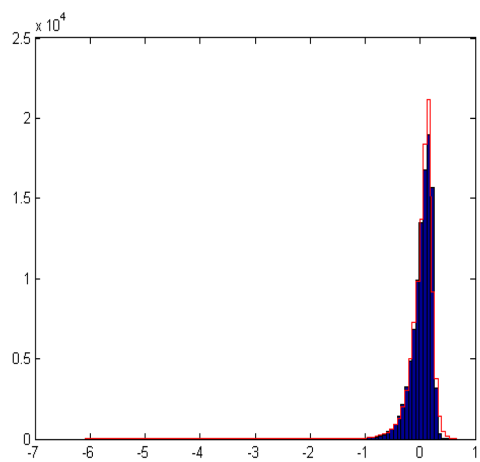
Figure 1: Comparison of estimated distribution by ET and given distributions



(a) Standard normal



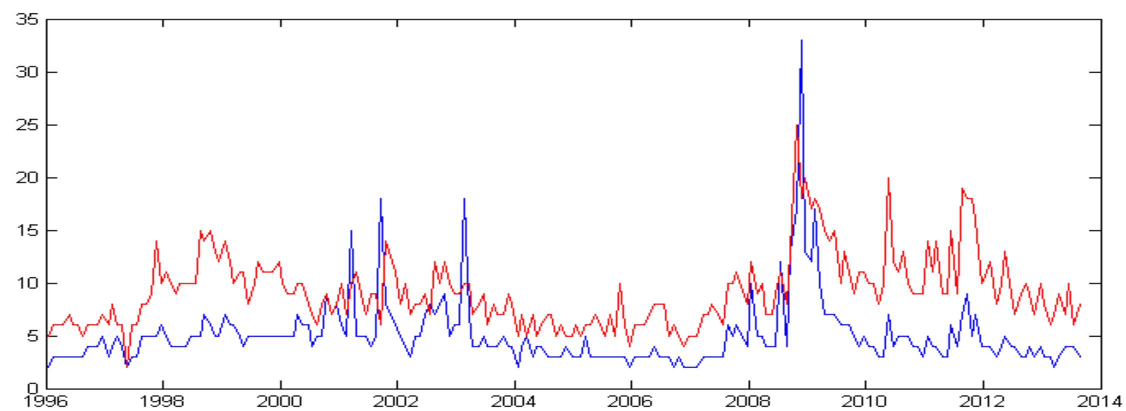
(b) Skewt(5,-0.3)



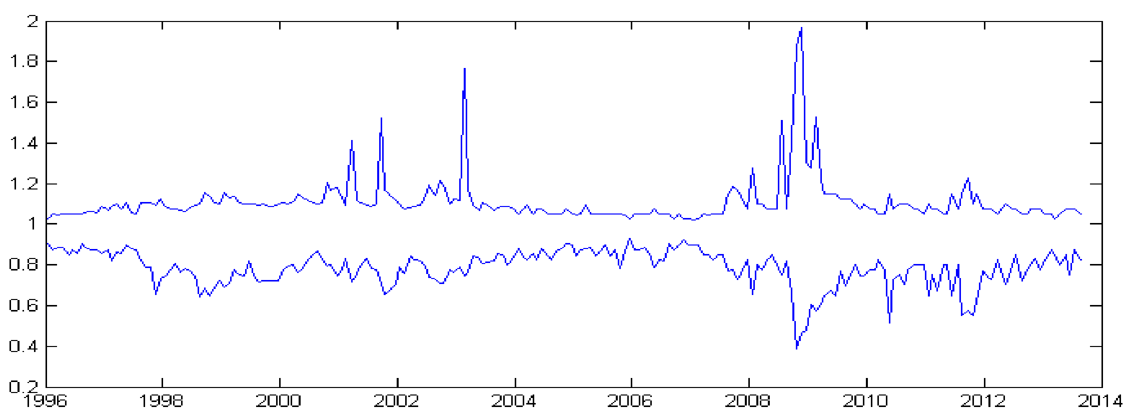
(c) Skewt(5,-0.7)

Note: The blue bars are histograms of the given risk neutral distributions of the continuous compounded return. The red lines are risk neutral densities estimated by principle of maximum entropy.

Figure 2: Option numbers and strike price range

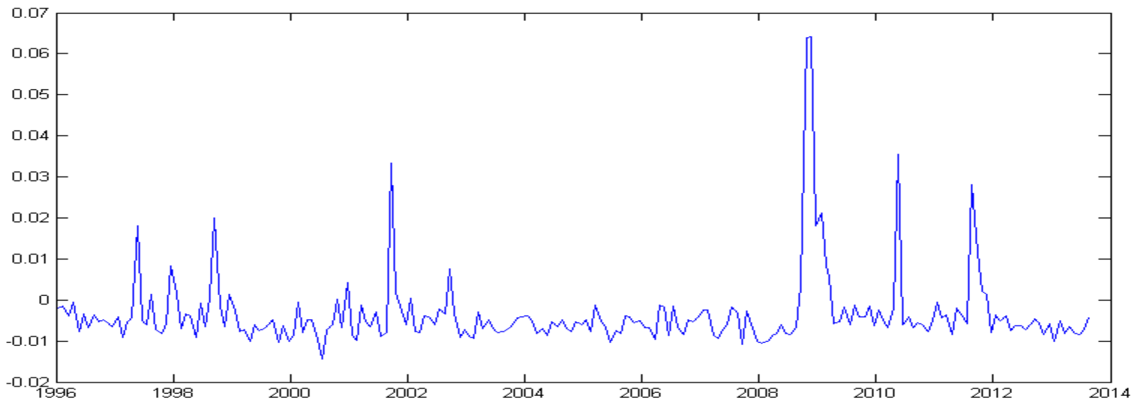


(a) Number of call and put options

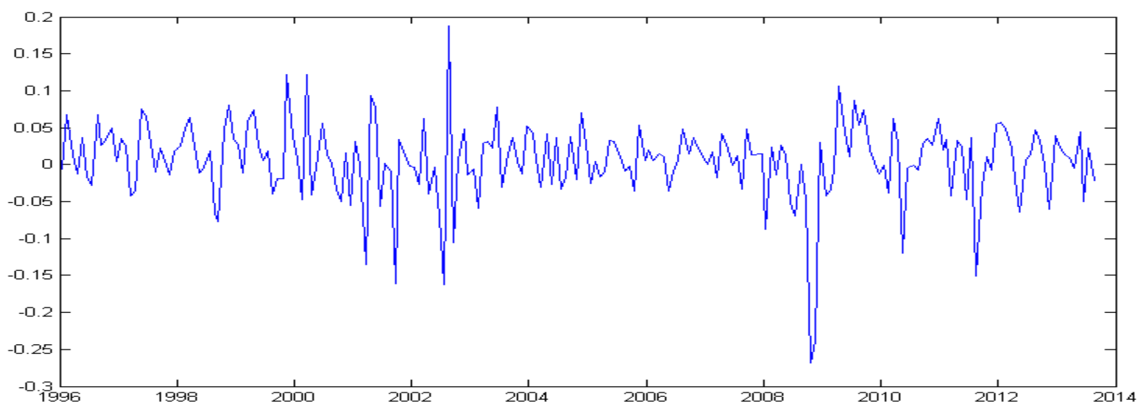


(b) Strike price range of the options

Figure 3: Comparison of MFIV and ETIV



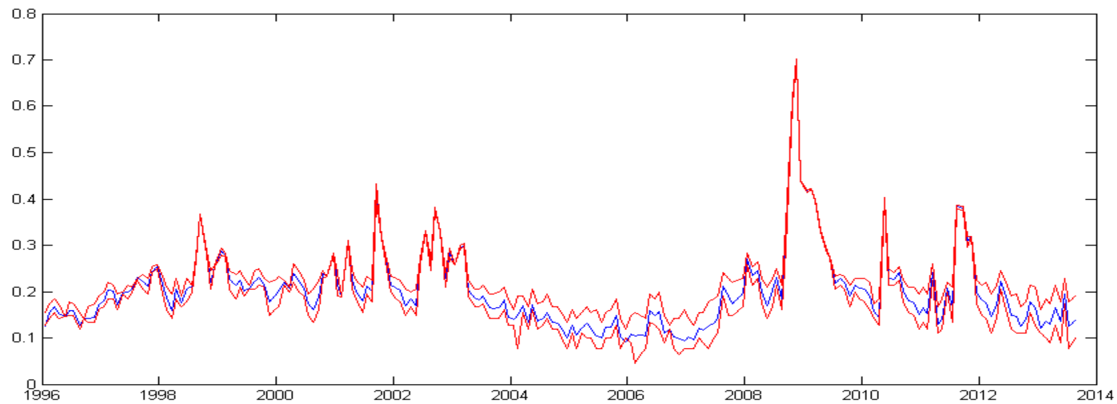
(a) Difference between ETIV and MFIV



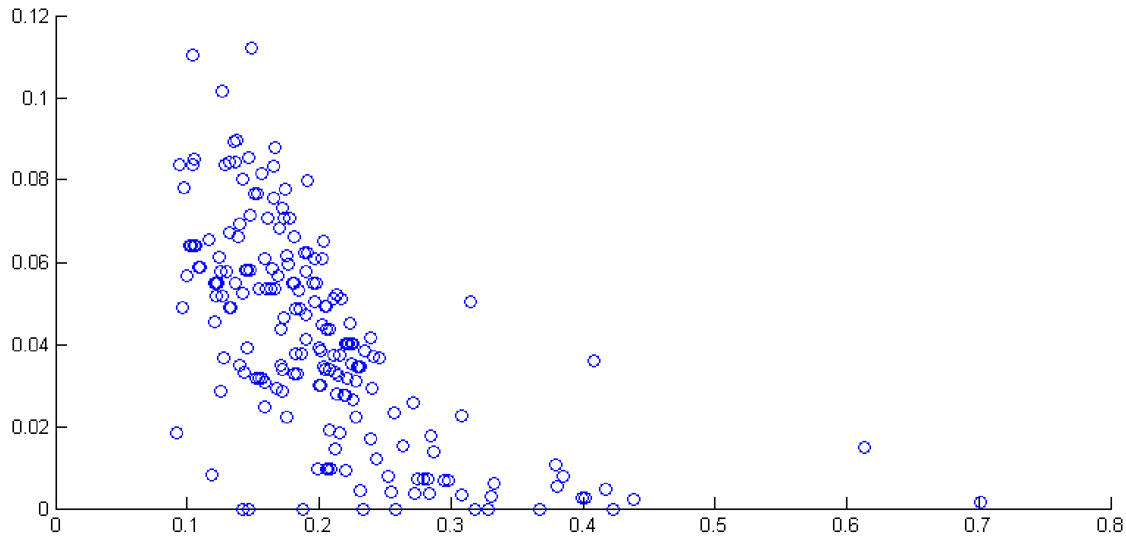
(b) S&P500 index return in the previous month

Note: Blue line is for put options and red line is for call options

Figure 4: Confidence interval for ETIV



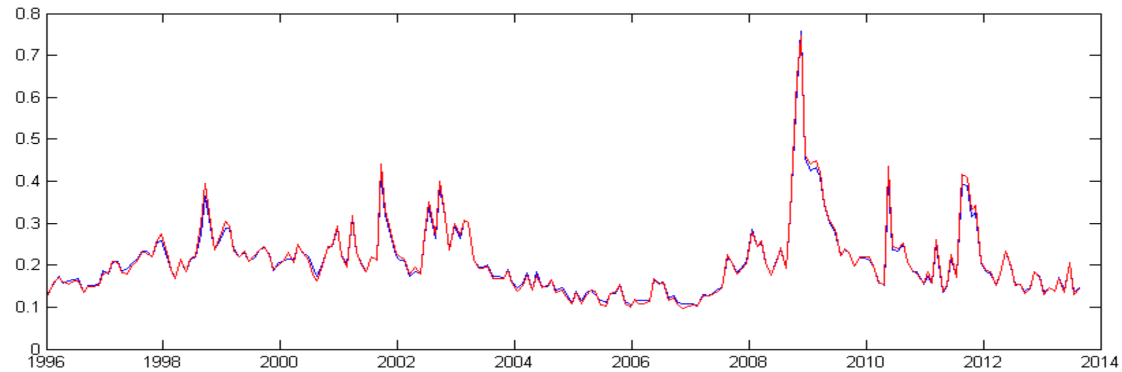
(a) Confidence interval for ETIV



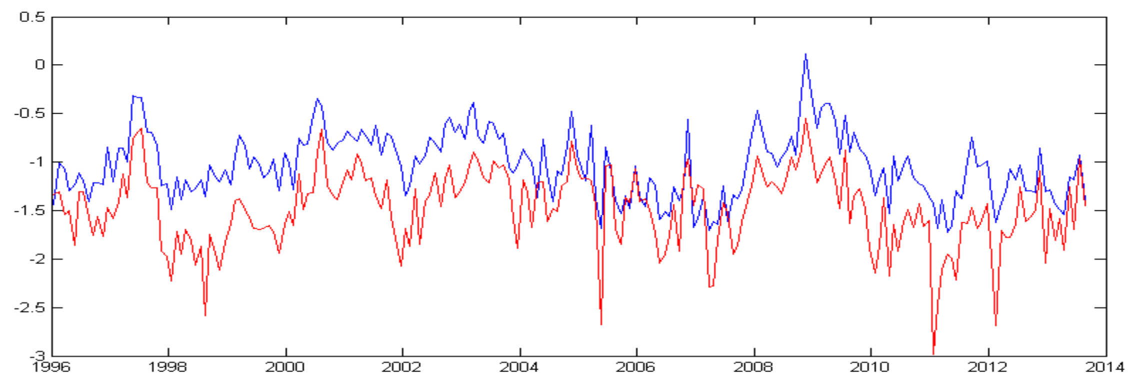
(b) ETIV and length of the confidence interval

Note: Red lines are the upper bound and lower bound of the confidence interval.

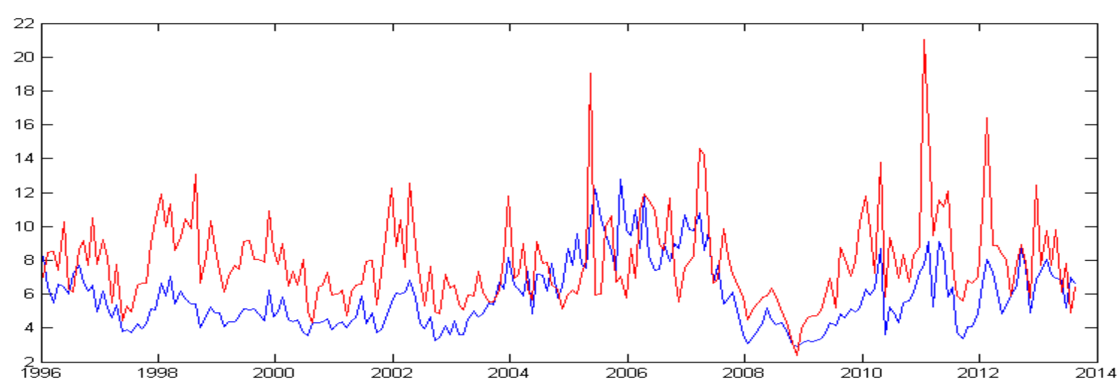
Figure 5: Risk neutral moments from S&P500 index options calculated by maximum entropy method and model-free method



(a) ETIV and MFIV



(b) ETIS and MFIS



(c) ETIK and MFIK

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